Scaling of core intrinsic rotation in ASDEX Upgrade L-mode plasmas using a robust regression technique

G. Verdooldaege¹,², R.M. McDermott², C. Angioni³ and the ASDEX Upgrade Team

¹Department of Applied Physics, Ghent University, 9000 Gent, Belgium
²Laboratory for Plasma Physics – Ecole Royale Militaire / Koninklijke Militaire School (LPP – ERM/KMS), 1000 Brussels, Belgium
³Max Planck Institute for Plasma Physics, Boltzmannstr. 2, 85748 Garching, Germany

INTRODUCTION

Plasma rotation and rotation shear play an important role in MHD stability and energy confinement. In current tokamaks plasma rotation is usually established through neutral beam momentum input, but this beam driven rotation is not expected to occur at a sufficient level in ITER. Therefore it is very important to determine whether sufficient intrinsic (or spontaneous) plasma rotation may still be established in ITER, which necessitates an improved understanding of the driving physical mechanisms. This work is aimed at clarifying the dependences of intrinsic toroidal rotation on local plasma parameters in a rotation database obtained in ASDEX Upgrade L-mode plasmas. To this end a powerful new regression method is deployed, called geodesic least squares (GLS), which is specifically targeted at estimating the nature and strength of relations in the presence of significant uncertainty in the regression model and in the measured response and predictor variables. We present the principles of GLS regression and show preliminary results of GLS rotation scalings, indicating the differences with ordinary least squares regression.

INRINCULAR TOROIDAL ROTATION IN ASDEX UPGRADE (AUG)

Intrinsic rotation in AUG L- and H-mode plasmas [1,2]:
- In H-mode, core rotation scales linearly with pedestal ion temperature gradient [3]
- In the presence of localized central electron heating the intrinsic rotation profile is determined by core-localized residualized profiles
- Both in L- and H-mode, core intrinsic rotation is highly correlated with the normalized toroidal rotation gradient $u'$ around mid-radius:
  - $u' \equiv \frac{\partial R}{\partial \Omega_{\psi}}$
- $u'$ depends on local plasma parameters
- $u'$ dependencies on dimensionless parameters are investigated in this study (other parameters may play a role):
  - Normalized logarithmic electron density gradient $\kappa/L_n$
  - Effective collisionality $v_{ce} \equiv \frac{v_0}{\nu}$
  - Normalized ion and electron temperatures gradients $R/L_n$ and $R/L_i$

EXPERIMENTAL TECHNIQUE

- Toroidal rotation profiles (122) from fast (4 ms) CXRS measurements in L-mode plasmas with a wide range of plasma parameters
- Minimize neutral beam torque via short (12 – 16 ms) neutral beam blips [4]
- Linear backwards extrapolation of velocity profile and its error bar to start of beam blip

![Figure 1: Evolution of the toroidal velocity profile during the 16 ms beam stop starting at 3.7 s in AUG pulse #28386.](image)

- Extrapolated profile (blue points in terms of $R$) is fitted with limited-flexibility model: 3rd order polynomial with design matrix $X$, coefficient vector $\beta$ and standard error $\sigma$, on predicted velocity $\dot{V}_{\psi}$ at new observation $R_{\psi}$:
  $$ X = \begin{bmatrix} 1 & R_s & R_{s}^2 & R_{s}^3 \end{bmatrix}, \quad \dot{V}_{\psi} = \begin{bmatrix} \dot{V}_{\psi}^\text{eff} \end{bmatrix}, \quad \sigma = \begin{bmatrix} \sigma \end{bmatrix}, \quad R_{\psi} = \begin{bmatrix} R_{\psi} \end{bmatrix}, \quad \beta \quad \text{SSE} = \| X \beta - \dot{V}_{\psi} \| ^2$$

- Spatial derivatives of profiles and their error bars $\dot{\sigma}$ are inherited from fitted model:
  $$ \frac{d\dot{V}_{\psi}}{dR} = \beta (X^T X)^{-1} X^T \sigma \left( \begin{array}{c} \dot{V}_{\psi} \\ \sigma \end{array} \right) \quad \dot{\sigma} = \sqrt{\frac{d\dot{V}_{\psi}}{dR} \frac{d\sigma}{dR}}$$

- Errors in derived quantities via Gaussian error propagation
- Electron density and temperature profiles from integrated analysis (IDA)
- IDA error bars via $\chi^2$ binning method: minimal dependence on prior [5]
- Assumption: relative error based on minimum IDA error bar:
  - 15% on $n_i$
  - 20% on $T_e$

![Figure 2: $n_i$ and $T_e$ profiles from IDA at 3.7 s in AUG pulse #28386.](image)

CONCLUSIONS

- Regression of local toroidal rotation gradient on key plasma parameters
- Considerable measurement and model uncertainties: need a robust regression method
- GLS more robust than OLS and estimates stronger effects of plasma profiles on toroidal rotation
- Future work: Determine most significant rotation predictors (model selection)
- Error bars on regression parameters using Bayesian geodesic regression

ACKNOWLEDGEMENT

This project has received funding from the EURATOM research and training programme 2014 – 2018.

REFERENCES


GEODESIC LEAST SQUARES REGRESSION

- Based on straightforward principles, generalization of ordinary least squares (OLS) [6,7]
- Outperforms established regression techniques, e.g. predicting L-H power threshold [7]
- Response to challenges for fusion scaling laws:
  - Large (non-Gaussian) uncertainties (noise) on response and predictor variables
  - Data outliers, heterogeneous data, heteroscedasticity
  - Logarithmic transformations affect error distributions
- Robust regression between probability distributions

Illustration: one predictor in linear model with Gaussian noise

- Theory: $\eta = \beta \cdot x$ experiment:
  $$ x = (x_1, x_2) \sim \mathcal{N}(0, \Sigma), \quad y = \beta^T x + \epsilon \sim \mathcal{N}(0, \Sigma_e) $$
  - With $n$ independent measurements $x_i$ and $y_i$, minimize difference between normal distributions
  $$ \rho_{OLS}(y_i | x_i) = \frac{1}{2} \exp \left( -\frac{(y_i - \beta^T x_i)^2}{2 \Sigma_e} \right) $$
  $$ \rho_{GLS}(y_i | x_i) = \frac{1}{2} \exp \left( -\frac{(y_i - \beta^T x_i)^2}{2 \Sigma} \right) $$
  - Estimate $\beta$ and $\sigma$

- Distance between probability density functions (PDFs): Rao geodesic distance (GD)
- Information geometry: PDF families $p(x|\eta)$ are Riemannian manifolds equipped with Fisher metric $g_{\eta}(\eta)$:
  $$ g_{\eta}(\eta) = -\frac{\partial^2 \ln p(x|\eta)}{\partial \eta \partial \eta^T}, \quad \text{e.g. Gaussian:} \quad g_{\eta}(\eta) = \frac{1}{\rho^2} \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \end{array} \right) $$

Figure 3: Rendering of the pseudosphere as a model for the univariate Gaussian manifold. Meridians are lines of constant mean, while circles of latitude have a constant standard deviation. Two geodesic curves are illustrated between some arbitrary points (distributions) on the manifold.

Rao geodesic distance (GD)

$$ D_{\text{G}}(\eta_1, \eta_2) = \int \sqrt{g_{\eta_1}(\eta_1) \cdot g_{\eta_2}(\eta_2)} \cdot \sqrt{h_{\eta_1}(\eta_1) \cdot h_{\eta_2}(\eta_2)} \cdot f(\eta_1) \cdot f(\eta_2) \, d\eta_1 \cdot d\eta_2 $$

Figure 4: Marginal scatter plot of $u'$ dependence on $\ln T_e$ ($\ln T_i$.

Conclusions:

- Ordinary least squares (OLS) regression yields the following scaling at $\rho_B = 0.35$ (in descending order of statistical significance) [2]:
  $$ u' = (0.12 \pm 0.02) \ln T_e - (0.09 \pm 0.04) \ln T_i - (0.06 \pm 0.03) \frac{1}{R_{\psi}} - (0.028 \pm 0.014) \frac{1}{R_{\eta}} $$

- Geodesic least squares (GLS) regression so far could not establish a meaningful scaling with all 4 predictors
- Regression on most significant variables $\ln T_e$ and $\ln T_i$:
  - OLS: $u' = -0.24 \ln T_e - 0.064 \ln T_i - 0.16 \ln T_i - 0.004 \frac{1}{R_{\psi}} - 0.064 \frac{1}{R_{\psi}}$
  - GLS: $u' = -0.35 \ln T_e - 0.17 \ln T_i - 0.028 \frac{1}{R_{\psi}} - 0.004 \frac{1}{R_{\psi}}$
  - Regression on $\ln T_e$ and $\ln T_i$:
    - OLS: $u' = -0.12 \ln T_e - 0.19 \ln T_i - 0.004 \frac{1}{R_{\psi}} - 0.004 \frac{1}{R_{\psi}}$
    - GLS: $u' = -0.19 \ln T_e - 0.22 \ln T_i - 0.004 \frac{1}{R_{\psi}} - 0.004 \frac{1}{R_{\psi}}$

Figure 5: Marginal scatter plot of $u'$ dependence on $\ln T_i$ ($\ln T_i$.

- Determine most significant rotation predictors (model selection)
- Error bars on regression parameters using Bayesian geodesic regression