The effect of electron-ion collisionality on ETG turbulence

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In electrostatic simulations of MAST plasma at electron-gyroradius scales, using the local flux-tube gyrokinetic code OS2 with adiabatic ions, we find that the long-time saturated electron heat flux (the level most relevant to energy transport) decreases as the electron collisionality decreases. At early simulation times the heat flux quasi-saturates at a level independent of electron collisionality; however the normal fluctuation component continues to grow slowly until much later times, eventually reducing the heat flux at low collisionality. We outline an explanation of this effect based on nonlinear-momentum interactions and the scaling of the zonal damping rate with electron-ion collisionality.

Improved energy confinement with decreasing collisionality has previously been observed on NSTX and MAST, and is favourable towards the performance of future devices.

INTRODUCTION

In STs there is experimental evidence for the dependence of energy transport on collisionality [1, 2]. In MAST, Volovik et al. found that the energy confinement time \(\tau_E = v_{\text{pol}}^3 \rho_{\text{pol}}^{-1} [\text{2}]\). Such a scaling is favorable towards improved confinement in future, hotter devices. There is also experimental evidence that NBI-heated MAST plasmas have high levels of sheared toroidal flow in which anomalous ion transport can be substantially suppressed [3]. Previous gyrokinetic simulations [4] support the paradigm that flow shear can stabilize otherwise unstable linear modes at ion gyroradius scales. Turbulence driven at electron gyroradius scales, in particular by the electron temperature gradient (ETG), may therefore govern confinement.

In this paper, we study the dependence of the electron heat flux \(Q_e\) on the electron collisionality \(\nu_e\) in a simple paradigm: with gyrokinetic electrons and hydrogenic adiabatic ions (AI) in the electrostatic limit (\(|\mathbf{B}| = 0\)). We will argue that \(Q_e \propto \nu_e^3\), the electron-ion collisionality, close to experimental values of \(R/L_T\), and that this scaling is associated with the collisional damping of zonal flows. We will show that at low \(\nu_e\), the slow evolution of the nonlinear turbulent state means that simulations of ETG turbulence must run to long times to reach saturation.

NUMERICAL SET-UP

We use the GSEQ code [5] to solve for the perturbed electron distribution function \(g = \langle e|F/T + h, \phi \rangle\), where \(e\) is the electron charge, \(\phi\) the perturbed electrostatic potential, \(T\) the electron temperature, \(F\) the background Maxwellian, \(\langle \cdot \rangle\) denotes the gyroaveraging operator (here at constant gyrocentre coordinate), and \(h\) satisfies the gyrostatic gyrokinetic equation (GKE)

\[
\frac{\partial h}{\partial t} + (v_e b + v_e) \cdot \nabla h + (\nabla E) \cdot (\nabla h + \nabla F) = \langle C[h] \rangle, \quad e \frac{\partial \langle \phi \rangle}{\partial t} = F.
\]

We drop the subscript \(e\) for electrons because this will cause no confusion. AI are equivalent to setting \(h_i = 0\), so that \(g_i = -e\phi|F/T\) only; taking ion and electron temperatures equal.

The quasi-neutrality condition then becomes simply \(2\nu_e v_i/T = -\nabla F|\phi|\). The integral is the perturbed electron density at particle coordinate \(r\).

We use the Abell-Barnes model collision operator \(\delta_f\), including electron-ion collisions. There is no drag offset term because \(h_i = 0 = \nu_i = 0\) (the parallel ion flow; cf. equation (135) of Ref. [6]). In \(k_y\) space:

\[
\langle C[h_i] \rangle = C_{GSEQ}[h_i] + \nu_i i k_y \left[ \frac{1}{2} \frac{\partial}{\partial \nu_i} \left( 1 - c_i^2 \right) \frac{\partial h_i}{\partial \nu_i} - \frac{1}{4} (1 + c_i^2) \frac{\partial^2 h_i}{\partial \nu_i^2} + \frac{1}{2} c_i^2 \frac{\partial h_i}{\partial \nu_i} \right].
\]

The two terms in the bracket are respectively the electron-ion pitch-angle scattering, and finite-Larmor-radius (FLR) diffusion. Analogous terms are present in the electron-electron piece of the operator, \(C_{GSEQ}\), which also includes correction terms so that it conserves particle number, momentum and energy. Electron-ion collisions relax the electron-ion pitch-angle scattering, \(\nu_i\), and finite-Larmor-radius (FLR) diffusion. Analogous terms are present in the electron-electron piece of the operator, \(C_{GSEQ}\), which also includes correction terms so that it conserves particle number, momentum and energy. Electron-ion collisions relax the electron-ion pitch-angle scattering, \(\nu_i\), and finite-Larmor-radius (FLR) diffusion. Analogous terms are present in the electron-electron piece of the operator, \(C_{GSEQ}\), which also includes correction terms so that it conserves particle number, momentum and energy.

The growth rate increases as collisionality is decreased, narrowing the stable region; the lowest \(k_y^2\) is unstable at the lowest collisionality shown here. At high \(k_y^2\), the growth rate decreases with collisionality. Except at the lowest and highest \(k_y^2\)'s and especially at the mid-range \(k_y^2\)'s which dominate the nonlinearly saturated heat flux – the fractional change in growth rate is small compared to the order-of-magnitude change in collisionality. We conjecture that these modest changes in the linear spectrum are not the cause of large changes in nonlinear saturation over the same range of collisionality.

HEAT FLUX SCALING WITH COLLISIONALITY

Figure 2 shows the variation of the time-averaged electron heat flux with collisionality and temperature gradient. The reduction in heat flux with collisionality can clearly be seen.

ZONAL FLOWS

Figure 3 shows the heat flux and the zonal potential squared versus time for points on the blue curve in Figure 2, \(a/L_T = 3.3\). After the initial linear growth phase, the heat flux quasi-saturates non-linearly. However, the zonal potential continues to grow slowly, leading to an eventual fall in the heat flux (starting at \(t = 50/\nu_i\)). Prior to this time, the evolution is statistically indistinguishable for the different collisionalities. However, the final saturated level after the fall is a function of collisionality (as shown in Figure 2). The onset time and the rate of fall are also functions of collisionality, but converge as the collisionality is reduced: the lowest three collisionalities in Figure 3 overlaid each other.

TABLE I. Nominal local equilibrium parameters (based approximately on MAST shot 8500 at \(t = 0.289\) s). \(a = 0.55\) m is the minor radius of the LCFS, the macroscopic normalizing length. The heat flux \(Q\) will be normalized by \(Q_{\text{GSEQ}} = n_e T_e (2\nu_e \rho_e)^2\), where \(\rho_e\) is the thermal ion gyroradius, the macroscopic normalizing length. In electron units \(\nu_e T_e = 0.02\nu_i T_i/\rho_i\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor radius</td>
<td>(r/a)</td>
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<tr>
<td>Major radius</td>
<td>(R/a)</td>
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<tr>
<td>Safety factor</td>
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<tr>
<td>Iprim (I_{iprim})</td>
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</tr>
<tr>
<td>Tprim (T_{iprim})</td>
<td>(3.42)</td>
</tr>
<tr>
<td>Collisionality (\nu_e/\nu_{\text{pol}})</td>
<td>(1.39)</td>
</tr>
</tbody>
</table>

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FIG. 1. Linear growth rates versus \(k_y^2\) at \(k_y = 0\) (with zero flow shear), for several collisionalities. (\(\theta = 0\) is the on-axis midplane, \(\theta = \pi\) the inboard midplane.) The spatial grid has \(N_y = 48\) cells in the parallel direction, and \(80 \times 108\) points in the perpendicular \(x \times y\) directions, in which the cross-section is square with minimum and maximum positive \(k_y\) and \(\nu_e/\nu_i\) grid points, respectively. There are \(N_y = 18\) energy grid points, and \(N_y = 16\) pitch-angles for passing particles in each direction. There is one pitch-angle for trapped particles bouncing at each \(\theta\) grid point. Note that, because of the twist-and-shift parallel boundary conditions in flux-tube geometry [7, 8], some of the lowest \(k_y\) 's have longer linked domains than the single \(2\pi\) range of \(\theta\).

FIG. 2. Variation of the time-averaged electron heat flux with collisionality, at the nominal temperature gradient, \(a/L_T = 3.3\), and one temperature gradient on either side.
In the GKE for a typical zonal mode, there are only nonzonal-nonzonal interactions, and no linear drive (since $\nabla F$ is in the $x$ direction). In this case, we can balance the nonlinear term against collisional damping, giving

$$k_i k_x \phi_{Z0} h_{Z0}/\nu \sim v_x h_{Z0}. \quad (4)$$

We use subscripts $Z, NZ$ to denote zonal and nonzonal respectively. Taking $h_{Z0}/\phi_x \sim h_{Z0}/\nu \sim eF/T$ and combining equations (3) and (4) we get

$$\frac{h_{Z0}}{\nu} \sim \frac{\phi_{Z0}}{\nu} \sim \frac{eF}{v_x R/L} \sim \frac{\nu eF}{(v_x/R\nu)^2}. \quad (5)$$

Alternatively, if the nonlinear term acts as a source at high frequency $\nu \gg v_x$, then equation (4) will be replaced by a scaling

$$k_i k_x \phi_{Z0} h_{Z0}/\nu \sim eF/T, \quad (6)$$

and the explicit scaling of the heat flux with collisionality will be $Q \propto \sqrt{\nu}$ instead.

**RELATIONSHIP TO EARLIER WORK**

At large enough $R/L$ or $v_x/h$, the zonal flow does not rise to a level high enough to produce the long-time fall in heat flux, which therefore remains at its ‘quasi-saturated’ in that case truly saturated — level. This can be seen at the highest collisionality in Figure 3, and is also evident in the lack of collisionality dependence at the high-collisionality end of Figure 2. In the high-$R/L$ limit we may recover the ‘critical balance’ scaling obtained for ion-temperature-gradient (ITG) turbulence by Barnes et al. [9] (which did not involve collisionality).

The effect of zonal damping on the saturated level of turbulence has previously been considered for ITG [10], and by Kim et al. for ETG [11]; in the large-aspect-ratio limit, they found that (as for the ITG case [12, 13]) the zonal damping time is shorter than the electron collision time. At the aspect ratio of order unity considered in the present paper, we find that the nonlinear state evolves on a time scale longer than the collision time.

Long-time changes in the saturated state of gyrokinetic simulations have been seen by others [14-16]. Some previous ETG results may be only ‘quasi-saturated’, but further exploration in parameter space is necessary to determine the full scope of these effects.

Microtearing has also been suggested as a possible explanation for the experimental transport scaling in STs [17, 18]. While microtearing is an inherently electromagnetic effect and cannot explain electrostatic results, we note that the theoretical arguments in the present paper do not depend explicitly on the nature of the instabilities driving the turbulence. We leave a more detailed consideration of this issue to future work.

**CONCLUSIONS**

In summary, we find that ETG turbulence simulations at driving gradients close to experimental levels and at low collisionality must be run to long times to capture the effect of zonal flows on the saturated state, and at these long times we find that heat transport $Q$ decreases with collisionality $\nu$. This is favorable towards improved confinement in future (hence, less collisional) devices such as CTF [19]. We have explained this behavior theoretically by balancing nonlinear and drive or damping terms in the GKE for nonzonal and zonal modes.

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